האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

המכון הישראלי ללימודים מתקדמים
Israel Institute for Advanced Studies

Random Walks on Symmetric Structures
Abstracts

Sunday, Oct 7

Tom Hutchcroft (Cambridge): **No percolation at criticality for groups of intermediate growth with fast heat kernel decay.**

I will prove that there is no percolation at criticality on groups of intermediate growth satisfying a heat kernel bound of the form $p_n(v,v) = e^{(-c\, n^{\gamma})}$ for $\gamma > 1/2$. Joint work with Jonathan Hermon.

Doron Puder (TAU): **Aldous' Spectral Gap Conjecture for Normal Sets.**

Aldous' Spectral gap conjecture, proved in 2009 by Caputo, Liggett and Richthammer, states the following a priori very surprising fact: the spectral gap of a random walk on a finite graph is equal to the spectral gap of the interchange process on the same graph.
This seminal result has a very natural interpretation in Representation Theory, which leads to natural conjectural generalizations. In joint works with Gadi Kozma and Ori Parzanchevski we study some of these possible generalizations, clarify the picture in the case of normal generating sets and reach a more refined, albeit bold, conjecture.

Gabor Elek (Lancaster): **Landscape with river.**

The action of a countable group on its Poisson boundary is always amenable in the sense of Zimmer. If the group is hyperbolic, then the Poisson boundary is the hitting measure on the Gromov boundary and the action of the group on the Gromov boundary is even boundary amenable. It is known however, that there exist groups (the Gromov-Osajda Monsters) which cannot act on a compact set in a boundary amenable fashion.

We prove that for any countable non-amenable group G, there exists a free, minimal, boundary amenable and purely infinite action of G on the non-compact Cantor set. This answers a question of Kellerhals, Monod and Rordam. The proof will be given using landscapes with rivers.

Dominic Yeo (Technion): **The critical window for random transposition random walk.**

The random walk on the permutations of [N] generated by the transpositions was shown by Diaconis and Shahshahani to mix with sharp cutoff around N log N /2 steps. However, Schramm showed that the distribution of the sizes of the largest cycles concentrates (after rescaling) on the Poisson-Dirichlet distribution PD(0,1) considerably earlier, after (1+\epsilon)N/2 steps. We show that this behaviour in fact emerges precisely during the critical window of (1+\lambda N^{-1/3}) N/2 steps, as \lambda \rightarrow\infty. Our methods are rather different, and involve an analogy with the classical Erdos-Renyi random graph process, the metric scaling limits of a uniformly-chosen connected graph with a fixed finite number of surplus edges, and analysing the directed cycle structure of large 3-regular graphs. Joint work with Christina Goldschmidt.

Monday, Oct 8

Yair Hartman (BGU): **Which groups have bounded harmonic functions?**

Bounded harmonic functions on groups are closely related to random walks on groups. It has long been known that all abelian groups, and more generally, virtually nilpotent groups are "Choquet-Deny groups": these groups cannot support non-trivial bounded harmonic functions. Equivalently, their Furstenberg-Poisson boundary is trivial, for any random walk.
I will present a recent result where we complete the classification of discrete countable Choquet-Deny groups, proving a conjecture of Kaimanovich-Vershik. We show that any finitely generated group which is not virtually nilpotent, is not Choquet-Deny. Surprisingly, the key is not the growth rate of the group, but rather the algebraic infinite conjugacy class property (ICC). This is joint work with Joshua Frisch, Omer Tamuz and Pooya Vahidi Ferdowsi.

Vadim Kaimanovich (Ottawa): **Spike decompositions and the associated boundaries.**

We shall discuss a new combinatorial structure on countable groups and the ensuing consequences for random walks. This is a joint work with Anna Erschler.

Yuval Peres (Microsoft Research): **Occupation measure of random walk in balls of Cayley graphs.**

Let G be a Cayley graph of an infinite, finitely generated group. Suppose that simple random walk on G is transient. Then the expected occupation time of a ball of radius r is at most $O(r^{2.5})$; it is an open problem whether the exponent 2.5 can be replaced by 2. I will explain how this relates to uniform spanning trees, dead ends in Cayley graphs, and estimating excursions between spheres. Joint work with Russell Lyons and Xin Sun.

Tuesday, Oct 9

Omer Angel (UBC): TBA
Vladas Sidoravicius (NYU Shanghai): TBA
Jeremie Brieussel (Montpellier): **Noise sensitivity of random walks on groups.**

Given a sample X_n of a random walk on a group, we consider its noised version which is a random word Y_n obtained from X_n after refreshing a given fixed proportion of increments. We say the random walk is noise sensitive if Y_n is close to an independent sample (independently of the probability to refresh). In this talk, I will present several notions of noise sensitivity, discuss their relationships and give a few examples. This is a joint work with Itai Benjamini.

Wednesday, Oct 10

Miklos Abert (Renyi Institute): **Kesten type theorems for groups, graphs and manifolds.**

Kesten, in his influential PhD thesis proved that for a normal subgroup N of Gamma, the visiting exponent (or, spectral radius of random walk) of N is greater than the visiting exponent of 1 if and only if N is amenable. In the talk we discuss generalizations of this result in the realm of finite and infinite graphs, invariant random subgroups and Riemannian manifolds, in particular, locally symmetric spaces. We also present some open problems. The results are due to Lyons-Peres, Gekhtman-Levit, Abert-Glasner-Virag, Mikolaj Fraczyk, Mustazee Rahman and Abert-Bergeron-Virag.

Daniel Luckhardt (BGU): **Benjamini-Schramm convergence of Riemannian manifolds.**

The concept of Benjamini-Schramm convergence can be extended to Riemannian manifolds or, more generally, to metric measure space. In this setup a question frequently studied is whether topological invariants that can be expressed as integers are continuous when normalized by the volume. An example of such an invariant is the Euler characteristic, that also exists for graphs. A vast generalization of the Euler characteristic for Riemannian manifolds are characteristic numbers. I will speak on my results showing continuity of normalized characteristic numbers on a suitable class of random Riemannian manifolds.

Ariel Yadin (BGU): **Intersectional IRSs and random walk entropy.**

Random walk entropy is a well studied concept with connections to many objects, most notably the Poisson-Furstenberg boundary and bounded harmonic functions. Our departure point is the question: what are the possible values for the random walk entropy (ranging over random walks on finitely generated groups)?
Although this question is still open, it leads to interesting connections with invariant random subgroups (IRSs) and Furstenberg entropy.

Nicolas Matte Bon (ETH Zürich): **Locally compact groups without invariant random subgroups and without uniformly recurrent subgroups.**

I will explain a way to construct examples of (non-discrete) locally compact groups that admit no non-trivial invariant random subgroups and no non-trivial uniformly recurrent subgroups. Some of them can be seen as (non-compactly generated) subgroups of the Neretin group of tree almost automorphisms. This is a joint work with Adrien Le Boudec.

Thursday, Oct 11

Romain Tessera (ENS): **A structure theorem for finite vertex-transitive graphs and applications to random walks.**

Using the theory of approximate groups, we prove a structure theorem for finite vertex transitive graphs with large diameter. We use it to prove various conjectures about growth, isoperimetry, resistance, and random walks on these graphs. This is joint work with Matthew Tointon, and partly with Itai Benjamini and Jonathan Hermon.

Ori Parzanchevski (HUJI): **Random walks on Ramanujan complexes and digraphs.**

In 2015, Lubetzky and Peres proved that random walk on Ramanujan graphs exhibits an optimal cutoff phenomenon. In joint work with Lubetzky and Lubotzky, we prove a similar result for certain random walks on Ramanujan complexes, which are a higher-dimensional generalization of Ramanujan graphs. The analysis involves a third notion, of Ramanujan digraphs (directed graphs), which gives a unified explanation for both low and high dimensional cases.

James Lee (UW): **Unimodular changes of metric.**

A nice "conformal" change of metric on a unimodular random graph can be used to analyze its spectral properties. In many instances, this can be used to establish a.s. recurrence, estimate the Green function, or bound the speed of the random walk. Gwynne and Hutchcroft recently employed this method (together with a change of metric derived from the LQG coupling) to confirm the precise speed exponent for UIPT. I will discuss the construction of such metrics, and some other applications, including a recent result with Ganguly, where we establish that the random walk on critical percolation in Z^2 is subdiffusive in the chemical distance (Kesten famously established subdiffusivity for the Euclidean distance, an a priori weaker estimate).

Alex Lubotzky (HUJI): **Can you hear the shape of a Cayley graph?**

The influential paper of M. Kac " Can you hear the shape of the drum?" (1966) have started a line of research studying, in various contexts (manifolds, graphs etc.) whether an object can be recovered from the spectrum of its Laplacian. Babai initiated this study for Cayley graphs.
We will show that simple groups like SL(n,q) have two systems of bounded size generators which give rise to isospectral non isomorphic Cayley graphs.
The seemingly easier problem for Sym(n) is still open.
The proof uses some machinery from the theory of automorphic forms and Mostow strong rigidity (but all these notions will be explained). Based on a joint work with B. Samuel and U. Vishne.